ENGR 0020
Spring 2019
Quiz 4
January 31, 2019
Time Limit: 10 Minutes

Name (Print): ______________________________
Recitation Section: __ (**A** 9:00-10:50, **B** 13:00-14:50)

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This quiz contains 1 page and 1 problem. You can use textbooks, notes and calculators, but *no* discussions. Use the backside of the paper if needed.

1. (10 points) Assume that two random variables (X, Y) are uniformly distributed on a circle with radius a. Then the joint probability density function is:

$$f(x,y) = \begin{cases} \frac{1}{2a^2}, & |x| + |y| \le a, \\ 0, & \text{otherwise.} \end{cases}$$

Find σ_X^2, the variance of X.

Solution: We have shown that $\mu_X = 0$ in the recitation. By $\sigma_X^2 = \mathbb{E}(X^2) - \mu_X^2$, we get $\sigma_X^2 = \mathbb{E}(X^2)$ in our case.

$$\begin{aligned} \mathbb{E}(X^2) &= \int_{-a}^{a} \frac{1}{2a^2} \, \mathrm{d}y \int_{-(a-|y|)}^{a-|y|} x^2 \, \mathrm{d}x \\ &= \frac{1}{3a^2} \int_{-a}^{a} (a - |y|)^3 \, \mathrm{d}y \\ &= \frac{2}{3a^2} \int_{0}^{a} (a - y)^3 \, \mathrm{d}y \qquad \text{by symmetry over } y \\ &= \frac{2}{3a^2} \int_{0}^{a} t^3 \, \mathrm{d}t \qquad \text{let } t = a - y \\ &= \frac{a^2}{6} \end{aligned}$$