ENGR 0020
Spring 2019
Quiz 5
February 10, 2019
Time Limit: 15 Minutes

Name (Print): ____________________
Recitation Section: __ (**A** 9:00-10:50, **B** 13:00-14:50)

Teaching Assistant: Shaoning Han

This quiz contains 1 page and 1 problem. You can use textbooks, notes and calculators, but *no* discussions. Use the backside of the paper if needed.

1. (10 points) The joint probability density function of two random variables X and Y is:

$$f(x,y) = \begin{cases} Cxy, & 0 \le x \le 1, 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Find C.

   (b) Are X and Y independent?

   (c) Let $Z = X - 3Y + 3$. Find $\mathbb{E}[Z]$ and $Var[Z]$.

Solution:

(a) Because

$$1 = \int_0^1 \int_0^1 f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int_0^1 \int_0^1 Cxy\,\mathrm{d}x\,\mathrm{d}y = \frac{C}{4},$$

we have $C = 4$.

(b) Yes.

(c) First, we have

$$\mathbb{E}[X] = \mathbb{E}[Y] = \frac{2}{3},$$

and

$$Var[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{1}{18}, \quad Var[Y] = Var[X] = \frac{1}{18}.$$

It follows that

$$\mathbb{E}[Z] = \mathbb{E}[X - 3Y + 3] = \mathbb{E}[X] - 3\mathbb{E}[Y] + 3 = \frac{5}{3}.$$

Because X and Y are independent, $Cov[X,Y] = 0$.
Hence,

$$Var[Z] = Var[X] + 9Var[Y] = \frac{5}{9}.$$